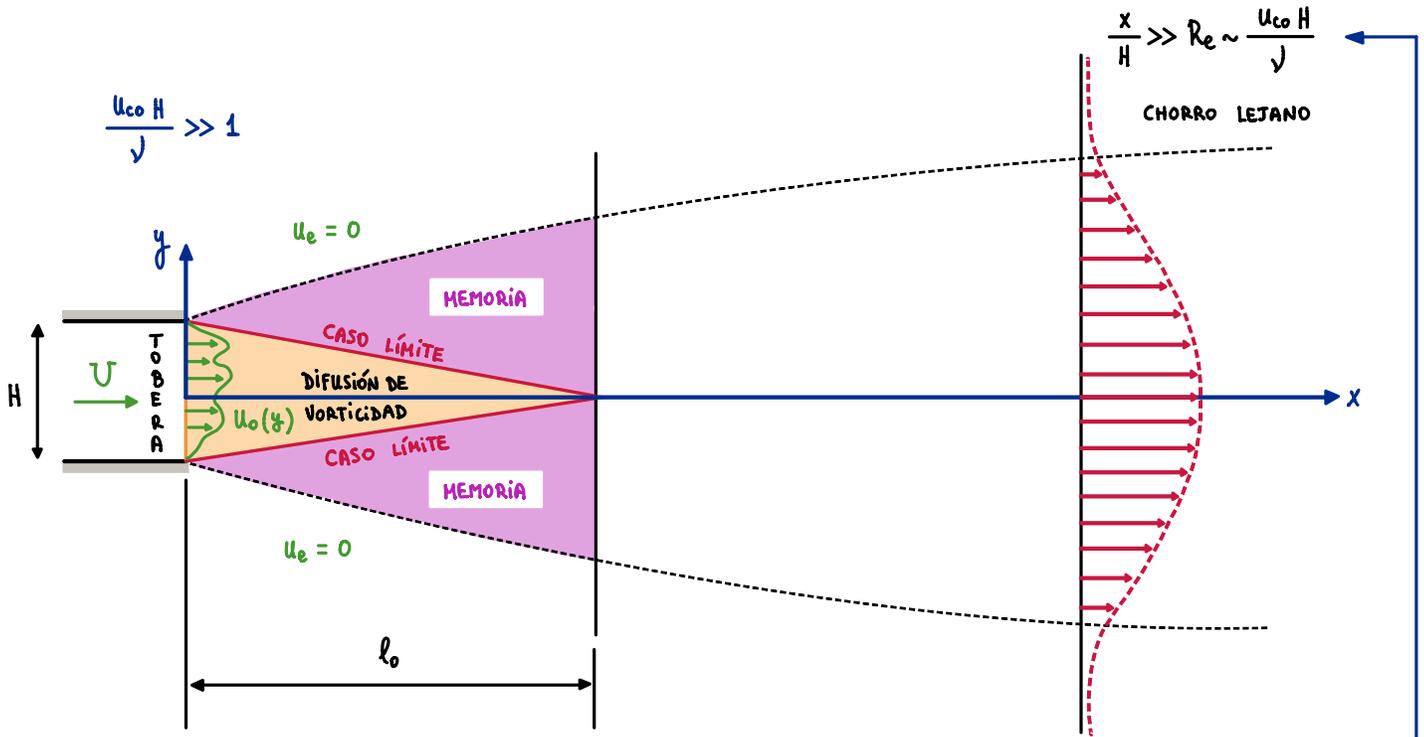


Chorro Plano Laminar



EFFECTOS VISCOSES : encargado de la "pérdida de memoria" de la tobera de salida.

$$ECdM_x : \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\sim \frac{U_0^2}{l_0}} = -\frac{\partial}{\partial x} \left(\frac{p}{\rho} + U_m \right) + \underbrace{\nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)}_{\sim \nu \frac{U_0}{H^2}}$$

Iguualando órdenes de magnitud de los términos CONVECTIVO y VISCOZO :

$$\frac{U_0^2}{l_0} \sim \nu \frac{U_0}{H^2} \longrightarrow \frac{l_0}{H} \sim \frac{U_0 H}{\nu} \sim Re \gg 1$$

$\frac{U_0 H}{\nu} \sim Re \gg 1$

$l_0 \sim H Re$

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pues la respuesta viscosa principal es po "11"

$\frac{\partial u}{\partial x} \sim \frac{U_0}{l_0}$ $\frac{\partial u}{\partial y} \sim \frac{U_0}{H}$ $\frac{\partial^2 u}{\partial x^2} \sim \frac{U_0}{l_0^2}$ $\frac{\partial^2 u}{\partial y^2} \sim \frac{U_0}{H^2}$

Condiciones de contorno:

$$\blacksquare \text{ Simetría: } \begin{cases} \blacksquare v(y) = v(-y) \longrightarrow y=0: v=0 \\ \blacksquare u(y) = u(-y) \longrightarrow y=0: \Delta_y u = 0 \longrightarrow y=0: \frac{\partial u}{\partial y} = 0 \end{cases}$$

$$\blacksquare y \rightarrow \infty: u \rightarrow u_e = 0$$

■ Falta la c.c. en "x" para "u" \longrightarrow Imponemos que en una estación $x^*/x^* \gg H \text{ Re}$

Conocemos u: $u(x^*, y) = u^*(y)$ ES PARTE DE LA SOLUCIÓN \longrightarrow DIFICULTAD

Si escribimos la ECDH_x en forma conservativa e integramos en la dirección transversal:

$$\frac{\partial}{\partial x} (u^2) + \frac{\partial}{\partial y} (u \cdot v) = \frac{\partial}{\partial y} \left(\nu \frac{\partial u}{\partial y} \right)$$

$$\int_0^{\infty} \frac{\partial}{\partial x} (u^2) dy + \int_0^{\infty} \frac{\partial}{\partial y} (u \cdot v) dy = \int_0^{\infty} \frac{\partial}{\partial y} \left(\nu \frac{\partial u}{\partial y} \right) dy$$

$$\frac{d}{dx} \int_0^{\infty} u^2 dy + \underbrace{u \cdot v \Big|_0^{\infty}}_0 = \nu \underbrace{\frac{\partial u}{\partial y} \Big|_0^{\infty}}_0$$

$$\frac{d}{dx} \int_0^{\infty} u^2 dy = 0$$

$$\int_0^{\infty} u^2 dy = I = \text{cte} \quad (\forall x \in \text{chorro lejano})$$

Para que valga $\forall x$ integramos mejor entre $-\infty$ y $+\infty$:

$$\frac{d}{dx} \int_{-\infty}^{\infty} u^2 dy + \underbrace{u \cdot v \Big|_{-\infty}^{\infty}}_0 = \nu \underbrace{\frac{\partial u}{\partial y} \Big|_{-\infty}^{\infty}}_0$$

$$\frac{d}{dx} \int_{-\infty}^{\infty} u^2 dy = 0$$

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$$I = \frac{1}{2} \int_{-\infty}^{\infty} u_0^2(y) dy = \frac{U^2 H}{2}$$

$$x \sim H \operatorname{Re} \sim \frac{U^2 H}{\nu}$$

El chorro tiene un núcleo central no viscoso, limitado por una capa de cortadura que va incorporando líquido del exterior al chorro.

No HAY SOLUCIÓN DE SEMEJANZA pues U y H intervienen por separado.

$$x \gg \frac{U^2 H}{\nu}$$

Los efectos viscosos afectan ya a todo el chorro y se pierde el detalle de lo que ocurre cerca de la tobera.

SOLUCIÓN DEL CAMPO LEJANO: interviene sólo I , no U y H por separado.

Eliminamos el parámetro ν de la ECDM_x: $x, \frac{x}{\sqrt{\nu}}, u, \frac{v}{\sqrt{\nu}}$

$$\frac{\partial u}{\partial x} + \frac{\partial \left(\frac{v}{\sqrt{\nu}} \right)}{\partial \left(\frac{x}{\sqrt{\nu}} \right)} = 0$$

$$u \frac{\partial u}{\partial x} + \frac{v}{\sqrt{\nu}} \frac{\partial u}{\partial \left(\frac{x}{\sqrt{\nu}} \right)} = \frac{\partial^2 u}{\partial \left(\frac{x}{\sqrt{\nu}} \right)^2}$$

$$\frac{x}{\sqrt{\nu}} = 0 : v = 0 ; \frac{\partial u}{\partial \left(\frac{x}{\sqrt{\nu}} \right)} = 0$$

$$\frac{x}{\sqrt{\nu}} \rightarrow \infty : u \rightarrow 0$$

$$\forall x: \int_0^{\infty} u^2 d \left(\frac{x}{\sqrt{\nu}} \right) = \frac{I}{\sqrt{\nu}}$$

VAR. INDEPENDIENTES: $x, \frac{x}{\sqrt{\nu}}$

VAR. DEPENDIENTES: $u, \frac{v}{\sqrt{\nu}}$

PARÁMETROS: $\frac{I}{\sqrt{\nu}}$

$$\longrightarrow u = u \left(x, \frac{x}{\sqrt{\nu}}, \frac{I}{\sqrt{\nu}} \right) ; \frac{v}{\sqrt{\nu}} = \frac{v}{\sqrt{\nu}} \left(x, \frac{x}{\sqrt{\nu}}, \frac{I}{\sqrt{\nu}} \right)$$

Ecuaciones de dimensiones:

$$[x] = L$$

[u] [v]

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UNICO PARÁMETRO ÚNICA V.I.

$$\left[\frac{I}{\sqrt{\nu}} \right] = \frac{(L T^{-1})^2 L}{(L^2 T^{-1})} = L^2 T^{-3/2}$$

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Introducimos la función de corriente ψ :

$$u = \frac{\partial \psi}{\partial y} \longrightarrow u = \frac{\partial \left(\frac{\psi}{\sqrt{J}} \right)}{\partial \left(\frac{y}{\sqrt{J}} \right)} \qquad v = -\frac{\partial \psi}{\partial x} \longrightarrow \frac{v}{\sqrt{J}} = -\frac{\partial \left(\frac{\psi}{\sqrt{J}} \right)}{\partial x}$$

De acuerdo con las ecuaciones y condiciones de contorno, la solución es de la forma:

$$\frac{\psi}{\sqrt{J}} = F \left(x, \frac{y}{\sqrt{J}}, \frac{I}{\sqrt{J}} \right) \qquad \left[\frac{\psi}{\sqrt{J}} \right] = \frac{L T^{-1} L}{(L^2 T^{-1})^{1/2}} = L T^{-1/2}$$

Adimensionalizamos $\frac{y}{\sqrt{J}}$ y $\frac{\psi}{\sqrt{J}}$:

$$\frac{\frac{y}{\sqrt{J}}}{x^\alpha \left(\frac{I}{\sqrt{J}} \right)^\beta} \longrightarrow \frac{T^{1/2}}{L^\alpha L^{2\beta} T^{-3\beta/2}} \longrightarrow \begin{cases} \frac{1}{2} = -\frac{3\beta}{2} \\ \alpha + 2\beta = 0 \end{cases} \longrightarrow \begin{cases} \alpha = \frac{2}{3} \\ \beta = -\frac{1}{3} \end{cases} \longrightarrow \frac{\frac{y}{\sqrt{J}}}{x^{2/3} \left(\frac{I}{\sqrt{J}} \right)^{1/3}}$$

$$\frac{\frac{\psi}{\sqrt{J}}}{x^\alpha \left(\frac{I}{\sqrt{J}} \right)^\beta} \longrightarrow \frac{L T^{-1/2}}{L^\alpha L^{2\beta} T^{-3\beta/2}} \longrightarrow \begin{cases} -\frac{1}{2} = -\frac{3\beta}{2} \\ \alpha + 2\beta = 1 \end{cases} \longrightarrow \begin{cases} \alpha = \frac{1}{3} \\ \beta = \frac{1}{3} \end{cases} \longrightarrow \frac{\frac{\psi}{\sqrt{J}}}{\left(\frac{I}{\sqrt{J}} x \right)^{1/3}}$$

Por tanto:

$$\frac{\psi}{\sqrt{J}} = f \left[\frac{\frac{y}{\sqrt{J}}}{x^{2/3} \left(\frac{I}{\sqrt{J}} \right)^{1/3}} \right]$$

$$\frac{\psi}{\sqrt{J}} = f(\eta) \quad ; \quad \eta = \frac{y I^{1/3}}{x^{2/3} \sqrt{J}} \quad \text{VARIABLE DE SEMEJANZA}$$

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de continuidad se satisface automáticamente al introducir la función de corriente), lo que quiere decir

$$\psi = (I\nu x)^{4/3} f(\eta) \quad ; \quad \eta = \frac{\gamma I^{1/3}}{3(\nu x)^{2/3}}$$

$$\frac{\partial \eta}{\partial x} = -\frac{2}{3} \frac{\eta}{x}$$

$$\frac{\partial \eta}{\partial \gamma} = \frac{I^{1/3}}{3(\nu x)^{2/3}}$$

Sustituimos en la ECDM_x:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial \psi}{\partial y} = \frac{d\psi}{d\eta} \frac{\partial \eta}{\partial y} = (I\nu x)^{4/3} \underbrace{\frac{df}{d\eta}}_{f'} \frac{I^{1/3}}{3(\nu x)^{2/3}} \rightarrow u = \frac{I^{2/3}}{3(\nu x)^{4/3}} f'$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{1}{3} (I\nu)^{4/3} x^{-2/3} f - \frac{d\psi}{d\eta} \frac{\partial \eta}{\partial x} = -\frac{1}{3} \frac{(I\nu)^{4/3}}{x^{2/3}} f - (I\nu x)^{4/3} \frac{df}{d\eta} \frac{\partial \eta}{\partial x} =$$

$$= -\frac{1}{3} \frac{(I\nu)^{4/3}}{x^{2/3}} f - (I\nu x)^{4/3} f' \left(-\frac{2}{3} \frac{\eta}{x}\right) = -\frac{1}{3} \frac{(I\nu)^{4/3}}{x^{2/3}} f + \frac{2}{3} \frac{(I\nu)^{4/3}}{x^{2/3}} \eta f' \rightarrow$$

$$\rightarrow v = -\frac{(I\nu)^{4/3}}{3 x^{2/3}} \left\{ f - 2 \eta f' \right\}$$

$$\frac{\partial u}{\partial x} = \frac{I^{2/3}}{3 \nu^{4/3}} \frac{\partial}{\partial x} (x^{-4/3} f') = \frac{I^{2/3}}{3 \nu^{4/3}} \left[-\frac{1}{3} x^{-4/3} f' + x^{-4/3} f'' \left(-\frac{2}{3} \frac{\eta}{x}\right) \right] \rightarrow$$

$$\rightarrow \frac{\partial u}{\partial x} = \frac{I^{2/3}}{9 \nu^{4/3} x^{4/3}} \left\{ f' + 2 \eta f'' \right\}$$

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$$\frac{\partial^2 u}{\partial y^2} = \frac{I}{9 \nu^5 x^{5/3}} f''' \rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{I^{4/3}}{9 \nu^5 x^{5/3}} f'''$$

Sustituyendo:

$$-\frac{I^{2/3}}{3(\sqrt{x})^{4/3}} f' \frac{I^{2/3}}{9\sqrt{x}^{4/3} x^{4/3}} \left\{ f' + 2\eta f'' \right\} - \frac{(I\sqrt{x})^{4/3}}{3x^{2/3}} \left\{ f - 2\eta f' \right\} \frac{I}{9\sqrt{x}} f'' = \sqrt{x} \frac{I^{4/3}}{27(\sqrt{x})^{5/3}} f'''$$

$$-\frac{I^{4/3}}{27\sqrt{x}^{2/3} x^{5/3}} (f')^2 - \frac{I^{4/3}}{27\sqrt{x}^{2/3} x^{5/3}} 2\eta f' f'' - \frac{I^{4/3}}{27\sqrt{x}^{2/3} x^{5/3}} f f'' + \frac{I^{4/3}}{27\sqrt{x}^{2/3} x^{5/3}} 2\eta f' f'' = \frac{I^{4/3}}{27\sqrt{x}^{2/3} x^{5/3}} f'''$$

$$-(f')^2 - 2\eta f' f'' - f f'' + 2\eta f' f'' = f'''$$

$$f''' + f f'' + (f')^2 = 0$$

EDO 3^{er} ORDEN
(3 c.c. en η)

En cuanto a la compatibilidad de las condiciones de contorno:

$$y=0 : \begin{cases} v=0 \\ \frac{\partial u}{\partial y}=0 \end{cases} \longrightarrow \eta=0 : \begin{cases} f=0 \quad (1) \\ f''=0 \quad (2) \end{cases}$$

$$y \rightarrow \infty : u \rightarrow 0 \longrightarrow \eta \rightarrow \infty : f' \rightarrow 0 \quad (3)$$

Si necesitamos que $\int_0^\infty (f') d\eta$ esté acotada,

Se debe satisfacer $\eta \rightarrow \infty : f' \rightarrow 0$,

colapsando las condiciones (3) y (4)

$$\forall x : \int_0^\infty u^2 dy = I \longrightarrow \int_0^\infty \frac{I^{4/3}}{9(\sqrt{x})^{2/3}} (f')^2 \frac{3(\sqrt{x})^{2/3}}{I^{1/3}} d\eta = I \longrightarrow \int_0^\infty (f')^2 d\eta = 3 \quad (4)$$

Esta c.c. deshace la homogeneidad del problema

Por tanto, el problema queda:

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Hacemos una primera integral :

$$f''' + ff'' + (f')^2 = 0$$

$$f''' + (ff')' = 0$$

$$\int f''' d\eta + \int (ff')' d\eta = 0$$

$$f'' + ff' = C$$

$$\downarrow \eta=0 : f=f''=0$$

$$C = 0$$

$$f'' + ff' = 0$$

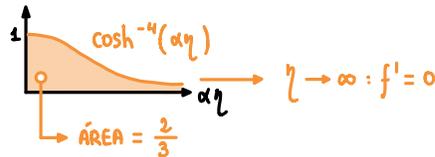
$$\eta=0 : f' = f'_0 / \int_0^{\infty} (f')^2 d\eta = 3$$

Integrando de nuevo :

$$f = 2\alpha \tanh(\alpha\eta) \quad f' = \frac{2\alpha^2}{\cosh^2(\alpha\eta)} \quad f'' = -\frac{4\alpha^3 \sinh(\alpha\eta)}{\cosh^3(\alpha\eta)}$$

El valor de "α" se obtiene de aplicar la condición de contorno que falta :

$$\int_0^{\infty} (f')^2 d\eta = 3 \rightarrow \frac{4\alpha^4}{\alpha} \int_0^{\infty} \frac{d(\alpha\eta)}{\cosh^4(\alpha\eta)} = 3 \rightarrow \frac{8}{3} \alpha^3 = 3 \rightarrow \alpha = \frac{9^{1/3}}{2}$$



Por tanto :

$$f = 9^{1/3} \tanh\left(\frac{9^{1/3}}{2} \eta\right)$$

$$u = \frac{I^{2/3}}{3(\nu x)^{1/3}} f'$$

$$9^{2/3} I^{2/3} \left(\frac{9^{1/3}}{2}\right)$$

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$$u = \frac{U^2 H}{2}$$

Flujo a través de una sección "x" del chorro :

$$\begin{aligned}
 q &= \int_{x=\text{cte}} \vec{v} \cdot \vec{n} \, dA = 2 \int_0^{\infty} u \, dy = \frac{C(\nu x)^{2/3}}{I^{1/3}} \int_0^{\infty} u \, d\eta = \frac{C(\nu x)^{2/3}}{I^{1/3}} \frac{(qI)^{2/3}}{3(\nu x)^{1/3}} \int_0^{\infty} \text{sech}^2\left(\frac{q^{1/3}}{2} \eta\right) d\eta = \\
 &= 2 \cdot q^{1/3} (\nu I x)^{1/3} \tanh\left(\frac{q^{1/3}}{2} \eta\right) \Big|_0^{\infty} = 2 \cdot (q \nu I x)^{1/3} \cdot (1 - 0) = 2 \cdot (q \nu I x)^{1/3}
 \end{aligned}$$

$$q = 2 \cdot (q \nu I x)^{1/3} \approx 4'1602 (\nu I x)^{1/3}$$

Dimensiones :

$$\begin{aligned}
 [q] &= [u][dy] = L T^{-1} L = L^2 T^{-1} \\
 ([\nu][I][x])^{1/3} &= (L^2 T^{-1} (L T^{-1})^2 L L)^{1/3} = (L^6 T^{-3})^{1/3} = L^2 T^{-1}
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} [q] &= [u][dy] \\ ([\nu][I][x])^{1/3} &= (L^2 T^{-1} (L T^{-1})^2 L L)^{1/3} \end{aligned}} \right\} \checkmark$$

Flujo en $x = 0$:

$$q_0 = q(x=0) = \int_{-\infty}^{\infty} u_0(y) \, dy = U H$$

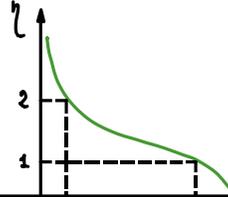
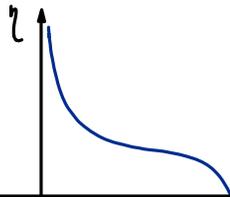
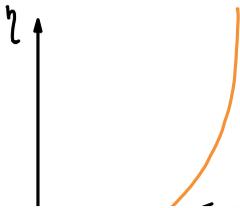
NO ES NULO OBIAMENTE

No sustituir $x=0$ en la expresión de $q(x)$ directamente pues la expresión de "u" utilizada sólo vale lejos

Entonces :

$$\frac{q}{q_0} = \frac{2 \cdot q^{1/3} \cdot \left(\frac{U^2 H}{2} \nu x\right)^{1/3}}{U H} = \left(\frac{36}{Re} \frac{x}{H}\right)^{1/3}$$

EL CHORRO ARRASTRA FLUIDO DESDE EL EXTERIOR Y LO INCORPORA A SU REGIÓN (ARRASTRE DEL CHORRO).



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por ejemplo

EL CHORRO ACABARÁ

$\uparrow x \rightarrow \uparrow Re$

SIENDO TURBULENTO